

Polarizing the dipoles

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Polarizing the dipoles

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ABSTRACT: We extend the massless dipole formalism of Catani and Seymour, as well as its massive version as developed by Catani, Dittmaier, Seymour and Trocsanyi, to arbitrary helicity eigenstates of the external partons. We modify the real radiation subtraction terms only, the primary aim being an improved efficiency of the numerical Monte Carlo integration of this contribution as part of a complete next-to-leading order calculation. In consequence, our extension is only applicable to unpolarized scattering. Upon summation over the helicities of the emitter pairs, our formulae trivially reduce to their original form. We implement our extension within the framework of HELAC-PHEGAS, and give some examples of results pertinent to recent studies of backgrounds for the LHC. The code is *publicly* available. Since the integrated dipole contributions do not require any modifications, we do not discuss them, but they are implemented in the software.

KEYWORDS: NLO Computations, Hadronic Colliders, QCD, Standard Model

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1 Introduction

At present, the necessity of next-to-leading order (NLO) calculations of QCD backgrounds for the Large Hadron Collider (LHC) is unquestionable. Much effort has been put into this problem, but until only recently, it seemed that the task is so huge that theory will stay behind the needs of experimentalists for quite some time. Whereas impressive calculations have been done with traditional methods based on Feynman diagrams [1–21], it is the new unitarity based methods [22–28] that provide some hope for accelerated progress. By now, there are three major groups with advanced software for virtual corrections [29–41], closely trailed by independent efforts [42]. Moreover, a full automate based on traditional methods is being built [43–46]. In any case, first successes have been recorded in [47–52] and look very promising.

Any NLO calculation consists of two parts, which are separately infrared (soft/collinear) divergent, the virtual corrections and real radiation. In order to allow for Monte Carlo simulations, two general classes of approaches have been devised, namely phase space slicing [53–58] and subtraction [59–65]. Currently, the latter approach seems to have proven its superiority. Its most accepted version, dipole subtraction, has been presented in the massless case in [62] and later for massive partons in [65] (see also [66]). There are a few completely automated implementations of this method, which have been presented in the literature [67–70], but only one is available for public use [70]. Moreover, it has often been criticized that the large number of terms in the dipole subtraction formalism is a practical

problem in realistic calculations, since it leads to a high computational costs. With the present publication we want to remedy both problems.

The basic idea is to allow for the same optimizations as those used in leading order simulations. The most important of these, besides phase space optimization, is the replacement of exact summation over external state polarizations by a probabilistic approach. In order not to ruin the Monte Carlo error estimates, the treatment must be consistent between the real emission contribution and the appropriate subtraction. This is the main subject of the present publication. Note that partial extensions of the dipole subtraction formalism to polarized states have been discussed before in [71, 72], but they were only concerned with scattering of polarized fermions or photons. The formulae from these studies are consistent with ours, when summed over the polarization of the gluon (or photon as discussed by these authors). Besides giving the appropriate formulae, we implement them within the framework of a fully fledged Monte Carlo generator, HELAC-PHEGAS [73–75], which has, on its own, already been extensively used and tested in phenomenological studies (see for example [76–79]). We demonstrate the potential of the software by performing some realistic simulations, in particular of the process $gg \rightarrow t\bar{t}b\bar{b}g$.¹ We also argue that the inclusion of the subtraction terms does not increase the total evaluation time per phase space point by large factors. In fact, the actual source of the substantially longer evaluation times in comparison to a leading order evaluation is the more complicated phase space, requiring orders of magnitude more accepted points to reach the same accuracy. In consequence, one can either further improve the evaluation time per phase space point by using color sampling for example (trivial in our approach), or concentrate on a better description of the phase space. We leave both tasks to future studies.

The paper is organized as follows. In the next section, we discuss the rôle and treatment of polarization in Monte Carlo generators, which leads us to the motivation for the present study. Subsequently, we describe the behavior of cross sections in soft and collinear limits, when polarized partons are present. Section 4 contains our main results, namely the dipole subtraction formulae for polarized external partons. Section 5, on the other hand, presents a few details of our implementation within HELAC-PHEGAS, as well as some realistic simulation results. We conclude in section 6.

2 Polarization in Monte Carlo simulations

Most practical problems, which are solved with Monte Carlo simulations involve unpolarized particles. In consequence, it is necessary to sum/average over the spin of the incoming and outgoing states. This increases the computational complexity of a calculation by a factor, which can, in principle, amount to $2^{n_2}3^{n_3}$, where n_2 and n_3 are the numbers of particles with 2 and 3 polarization states respectively. There are usually some symmetries, like the chiral symmetry in the massless fermion approximation, or supersymmetry in the pure gluon case, which reduce the number of degrees of freedom. Moreover, with modern recursive methods for tree level matrix element evaluation it is possible to reuse parts of

¹While this publication was being prepared, a complete study of the full hadronic process has been published [21].

a result to speed up the summation. Nevertheless, deterministic, exact methods have an inherent slow down factor, which cannot be completely removed.

Since the phase space integration is already done with probabilistic methods, and the polarization sum is not coherent, it is clearly desirable to replace this sum by some kind of random sampling as well. The approach, which is most often used is to sample over helicity. The main disadvantage here is that different helicity configurations contribute very differently to the final result. In fact, several orders of magnitude between contributions are usually observed. If we use a flat distribution to pick a helicity configuration, the final variance of a result will be increased by a substantial factor, not to mention the increase in the number of generated phase space points to obtain a given error estimate. The situation can be improved by using stratified sampling, which is usually also used for phase space optimization. One of the possible algorithms, which we implemented in our software, has been presented in [80] (more details will follow in section 5).

Another approach, which is substantially easier to implement has been proposed in [81, 82]. The idea is to replace summation over helicity by integration over a phase. For example, a gluon polarization state can be written as

$$\epsilon_\mu(k, \phi) = e^{i\phi} \epsilon_\mu(k, +) + e^{-i\phi} \epsilon_\mu(k, -) , \tag{2.1}$$

where $\epsilon_\mu(k, \pm)$ are helicity eigenstates. The sum over the helicity of this gluon fulfills the following identity

$$\sum_\lambda |M_\lambda|^2 = \frac{1}{2\pi} \int_0^{2\pi} d\phi |M_\phi|^2 . \tag{2.2}$$

Notice that the range of integration could have been reduced to $[0, \pi]$ with the same result. We keep 2π as the upper bound in order to accommodate the third degree of freedom of massive gauge bosons, which is then added in eq. (2.1) without any phase factor. Obviously, we do not expect large differences in the values of $|M_\phi|^2$, as function of ϕ , since for every value of ϕ we have both helicities contributing. In consequence, a flat distribution in the Monte Carlo sampling should provide satisfactory results, which has been confirmed on specific examples in [81, 82] and also in our studies.

At this point, we should decide which approach to chose for dipole subtraction. As we will show in the next section, helicity eigenstates provide particularly simple formulae for this problem, which are only minor modifications of the original formalism. Therefore, we trade the simplicity of the implementation of the Monte Carlo integration over general polarizations with a phase, for simple dipole subtraction formulae. Practice also shows that whereas helicity sampling is superior for a small number of helicity configurations, with many particles general polarizations with phases start to dominate. The reason is that at some point the algorithm of helicity sampling cannot find the optimal distribution. In our case, where the number of partons is relatively low, since we are always thinking of a next-to-leading order calculation, which is bound in complexity by our ability to compute virtual corrections, the large number of final states comes from electroweak decays into colorless states. We found it optimal to use a hybrid model, where partons have definite helicities and remaining particles have general polarizations with phases.

3 Soft and collinear limits for polarized partons

In view of the considerations of the previous section, we have two possibilities to treat the polarization of partons. Indeed, we can either use arbitrary polarization vectors, or helicity eigenstates. Let us first show that, when using the latter the soft limit is particularly simple.

It is well known that the exchange of a soft gluon between two partons (quarks or gluons) can be approximated by eikonal currents

$$|M_{m+1}|^2 \sim_m \langle \dots | \mathbf{J}^{\mu,a \dagger} \mathbf{J}_\mu^a | \dots \rangle_m, \quad (3.1)$$

where we have omitted irrelevant constants and the current \mathbf{J} is given by

$$\mathbf{J}^{\mu,a} = \sum_i \mathbf{T}_i^a \frac{p_i^\mu}{p_i k}, \quad (3.2)$$

with k the momentum of the soft gluon and p_i the momentum of a hard parton. The color charge operators \mathbf{T}^a are defined as in [62], i.e. for two given color space vectors $|a_1, \dots, a_m\rangle$ and $|b_1, \dots, b_n\rangle$, we have

$$\langle a_1, \dots, a_i, \dots, a_m | \mathbf{T}_i^c | b_1, \dots, b_i, \dots, b_m \rangle = \delta_{a_1 b_1} \dots T_{a_i b_i}^c \dots \delta_{a_m b_m}, \quad (3.3)$$

where $T_{bc}^a = if_{bac}$ (color charge matrix in the adjoint representation) if parton i is a gluon, $T_{\alpha\beta}^a = t_{\alpha\beta}^a$ (color charge matrix in the fundamental representation) if parton i is an outgoing quark, and $T_{\alpha\beta}^a = -t_{\beta\alpha}^a$ if parton i is an outgoing anti-quark. The charges of in-going partons are defined by crossing. With this definition

$$\sum_i \mathbf{T}_i^a = 0, \quad (3.4)$$

which means that no signs are needed in eq. (3.2), and the current is both transverse and self-conjugate

$$k_\mu \mathbf{J}^{\mu,a} = 0, \quad \mathbf{J}^{\mu,a \dagger} = \mathbf{J}^{\mu,a}. \quad (3.5)$$

While it is clear that the eikonal approximation, and thus also the soft limit, is independent of the polarization of the hard partons, if the soft gluon is polarized on the other hand, we have

$$\mathbf{J}^{\mu,a \dagger} \mathbf{J}_\mu^a \longrightarrow -\mathbf{J}^{\mu,a \dagger} \mathbf{J}^{\nu,a} \epsilon_\mu(k, \lambda) \epsilon_\nu^*(k, \lambda). \quad (3.6)$$

Crucially, for helicity eigenstates

$$\epsilon_\mu^*(k, +) = e^{i\psi} \epsilon_\mu(k, -), \quad (3.7)$$

where ψ is some phase, which can be freely chosen. With this relation and the properties of the eikonal current, it is easy to show that

$$\mathbf{J}^{\mu,a \dagger} \mathbf{J}^{\nu,a} \epsilon_\mu(k, +) \epsilon_\nu^*(k, +) = \mathbf{J}^{\mu,a \dagger} \mathbf{J}^{\nu,a} \epsilon_\mu(k, -) \epsilon_\nu^*(k, -) = \frac{1}{2} \mathbf{J}^{\mu,a \dagger} \mathbf{J}_\mu^a. \quad (3.8)$$

Thus, we have shown that the soft limit is independent of the helicity of the soft gluon. This is important, because this means that we can introduce helicity into the unpolarized

dipole subtraction formulae without explicit reference to the polarization vectors of the soft gluons. Unfortunately, for general polarization vectors, such as those defined in section 2, there is no relation of the type of eq. (3.7). In consequence, if we wanted to use general polarization vectors, a less trivial modification of the dipoles would be needed.

Let us now turn to the collinear limit. It is in principle possible to work directly with amplitudes, however due to the treatment of the soft limit, the original dipole subtraction formalism has been formulated for squares of matrix elements. Here we shall proceed similarly.

We consider the quasi-collinear limit for massive partons and the corresponding true collinear limit in the massless case. For a pair $\{i, j\}$ of outgoing partons, which become collinear, we assume the following momentum parameterization

$$p_i^\mu = zp^\mu + k_\perp^\mu - \frac{k_\perp^2 + z^2 m_{ij}^2 - m_i^2}{z} \frac{n^\mu}{2pn}, \quad (3.9)$$

$$p_j^\mu = (1-z)p^\mu - k_\perp^\mu - \frac{k_\perp^2 + (1-z)^2 m_{ij}^2 - m_j^2}{1-z} \frac{n^\mu}{2pn}, \quad (3.10)$$

where $p_i^2 = m_i^2$, $p_j^2 = m_j^2$ and $p^2 = m_{ij}^2$, n is a light-like auxiliary vector, and k_\perp is the transverse component orthogonal to both p and n , which parameterizes the collinear limit. The parton of mass m_{ij} is the virtual particle, which splits into i and j . Its nature is uniquely determined in QCD. For example, if i is a quark and j is an anti-quark, then $m_{ij} = 0$ corresponds to a virtual gluon. As in [65], we define the limit by a uniform rescaling

$$k_\perp \rightarrow \alpha k_\perp, \quad m_i \rightarrow \alpha m_i, \quad m_j \rightarrow \alpha m_j, \quad m_{ij} \rightarrow \alpha m_{ij}, \quad (3.11)$$

with $\alpha \rightarrow 0$. The matrix element behaves in this limit as

$$m_{+1} \langle \dots, \{p_i, \lambda_i\}, \dots, \{p_j, \lambda_j\}, \dots \mid \dots, \{p_i, \lambda_i\}, \dots, \{p_j, \lambda_j\}, \dots \rangle_{m+1} \underset{\alpha \rightarrow 0}{\sim} \quad (3.12)$$

$$\frac{1}{\alpha^2} \frac{8\pi\alpha_s}{(p_i + p_j)^2 - m_{ij}^2} m \langle \dots, \{p, \lambda'\}, \dots \mid \hat{P}_{\tilde{i}j, i}(z, k_\perp, \{m\}, \{\lambda\}) \mid \dots, \{p, \lambda\}, \dots \rangle_m,$$

where \hat{P} are generalized Altarelli-Parisi kernels (here in four dimensions), $\{m\}$ is the set of masses, and $\{\lambda\}$ is the set of helicities, whereas $\tilde{i}j$ is the emitter parton (we will call the original pair, the emitter pair).

Whereas the unpolarized massive case of \hat{P} has been presented in [65], the polarized massless case can be read to a large extent already from [83]. Here we present the formulae, which contain all the information

$$\langle \lambda' \mid P_{QQ}(z, k_\perp, m_Q, \lambda_Q, \lambda_g) \mid \lambda \rangle = \quad (3.13)$$

$$C_F \left\{ \frac{\delta_{\lambda\lambda_Q}(z^2 + \delta_{\lambda_Q\lambda_g}(1-z^2))}{1-z} - \delta_{\lambda\lambda_g} \delta'_{\lambda_Q\lambda_g} \frac{m_Q^2}{p_Q p_g} \right.$$

$$\left. - \left[\left(\delta_{\lambda\lambda_Q} \delta_{\lambda_Q\lambda_g} - \delta'_{\lambda\lambda_Q} \delta'_{\lambda_Q\lambda_g} \right) \frac{1}{z} + \delta'_{\lambda_Q\lambda_g} (\delta_{\lambda\lambda_Q} - \delta_{\lambda\lambda_g}) z \right] \frac{m_Q^2}{2p_Q p_g} \right\} \delta_{\lambda'\lambda};$$

$$\begin{aligned}
 \langle \lambda' | P_{gQ}(z, k_{\perp}, m_Q, \lambda_Q, \lambda_{\bar{Q}}) | \lambda \rangle = & \quad (3.14) \\
 T_R \left\{ \frac{\delta_{\lambda'\lambda} \delta'_{\lambda_Q \lambda_{\bar{Q}}} - 2\delta'_{\lambda_Q \lambda_{\bar{Q}}} (k_{\perp} \cdot \epsilon(p, \lambda'))^* (k_{\perp} \cdot \epsilon(p, \lambda))}{2} \right. \\
 & + \delta_{\lambda'\lambda} \left[\delta'_{\lambda_Q \lambda_{\bar{Q}}} \left(\delta_{\lambda\lambda_Q} z + \delta_{\lambda\lambda_{\bar{Q}}} (1-z) - \frac{1}{2} \right) \right. \\
 & \left. \left. + \left((\delta_{\lambda\lambda_Q} - \delta'_{\lambda_Q \lambda_{\bar{Q}}}) \frac{1}{z} + (\delta_{\lambda\lambda_{\bar{Q}}} - \delta'_{\lambda_Q \lambda_{\bar{Q}}}) \frac{1}{1-z} \right) \frac{m_Q^2}{(p_Q + p_{\bar{Q}})^2} \right] \right\} ;
 \end{aligned}$$

$$\begin{aligned}
 \langle \lambda' | P_{gg}(z, k_{\perp}, \lambda_i, \lambda_j) | \lambda \rangle = & \quad (3.15) \\
 C_A \left\{ \delta_{\lambda'\lambda} \left(\frac{\delta_{\lambda\lambda_i}}{1-z} + \frac{\delta_{\lambda\lambda_j}}{z} - 2\delta'_{\lambda_i \lambda_j} \right) - 2\delta'_{\lambda_i \lambda_j} z(1-z) \frac{(k_{\perp} \cdot \epsilon(p, \lambda'))^* (k_{\perp} \cdot \epsilon(p, \lambda))}{k_{\perp}^2} \right. \\
 & \left. + \delta_{\lambda'\lambda} [\delta_{\lambda\lambda_i} - \delta_{\lambda\lambda_j}] (1-2z) \right\} ;
 \end{aligned}$$

where $\delta'_{\lambda_i \lambda_j} = 1 - \delta_{\lambda_i \lambda_j}$. Notice that the contents of the square brackets vanish upon summation over the helicities of the emitter partons. Because the soft limit remains untouched with helicity eigenstates, the above formulae alone can be used to modify the dipole subtraction terms. An important difference to previous studies is that we do not use open Lorentz indices, but contract directly with polarization vectors. This is allowed by the transversality of the k_{\perp} vector (and its counterparts in the final dipole subtraction formulae) and of the matrix element. Moreover, we used the phase conventions for the polarization vectors as given in [73], which are also consistent with those of MADGRAPH [84]. Any difference in conventions should be relatively easy to compensate.

Notice finally, that the initial state splittings can be derived from the above formulae by crossing.

4 Dipole subtraction with helicity eigenstates

The dipole subtraction formalism has been described to great extent in [62] and [65]. We do not repeat this discussion and assume that the reader is familiar with the main concepts. On the other hand, we give all the necessary formulae for a complete implementation in a numerical program, i.e. not only the modified splitting kernels, but also the momentum remappings.

The starting point of our exposition is the subtracted real radiation contribution to a next-to-leading order cross section

$$\int d\Phi \sum (|M_{m+1}|^2 - \mathcal{D}), \quad (4.1)$$

where the sum runs over polarizations and colors, and an average over the initial state colors and polarizations is understood together with a symmetry factor for the final states. We have also omitted the jet functions in the phase space integration $d\Phi$, since they are

irrelevant at the moment. In this somewhat schematic expression, the dipole contribution can be decomposed as

$$\mathcal{D} = \sum_{\{i,j\}} \sum_{k \neq i,j} \mathcal{D}_{ij,k} + \sum_{\{i,j\}} \sum_a \mathcal{D}_{ij}^a + \sum_{a,i} \sum_{j \neq i} \mathcal{D}_j^{ai} + \sum_{a,i} \sum_{b \neq a} \mathcal{D}^{ai,b}, \quad (4.2)$$

where i, j, k denote final states, whereas a, b initial states. A pair of indices corresponds to the emitter pair and a single index specifies the spectator. The fact that the sum in eq. (4.1) runs over the difference of the real emission matrix element squared and the dipole subtraction contribution is not a coincidence of course, since we want to have a cancellation of divergences for each helicity configuration. The formulae that we present in the following guarantee that a contribution corresponding to a given helicity configuration of the partons is finite in soft and collinear limits. In principle, we could also give the formulae in a form, in which the finiteness would extend to a given color flow, but we refrain from this in the present work.

Clearly, in each of the dipoles in eq. (4.2) all the polarizations, but those of the emitter pair, must be taken over from the matrix element M_{m+1} . Since the sums run over all partons and our formulae are only valid for helicity eigenstates, we have to require that all partons be in helicity eigenstates. On the other hand, there is no restriction on the polarization states of the remaining particles.

Finally, let us stress that since we are only interested in the subtraction for the real emission, we work exclusively in four space-time dimensions.

4.1 Final state emitter and final state spectator

The single dipole contribution is

$$\begin{aligned} \mathcal{D}_{ij,k}(\dots, \{p_i, \lambda_i\}, \dots, \{p_j, \lambda_j\}, \dots, \{p_k, \lambda_k\}, \dots) &= -\frac{1}{(p_i + p_j)^2 - m_{ij}^2} \\ &\times \sum_{\lambda', \lambda} \langle \dots, \{\tilde{p}_{ij}, \lambda'\}, \dots, \{\tilde{p}_k, \lambda_k\}, \dots | \frac{\mathbf{T}_k \cdot \mathbf{T}_{ij}}{\mathbf{T}_{ij}^2} \mathbf{V}_{ij,k} | \dots, \{\tilde{p}_{ij}, \lambda\}, \dots, \{\tilde{p}_k, \lambda_k\}, \dots \rangle, \end{aligned} \quad (4.3)$$

where the momentum remapping is given by ($Q = p_i + p_j + p_k$)

$$\tilde{p}_k^\mu = \frac{\sqrt{\lambda(Q^2, m_{ij}^2, m_k^2)}}{\sqrt{\lambda(Q^2, (p_i + p_j)^2, m_k^2)}} \left(p_k^\mu - \frac{Q p_k^\mu Q^\mu}{Q^2} \right) + \frac{Q^2 + m_k^2 - m_{ij}^2}{2Q^2} Q^\mu, \quad (4.4)$$

$$\tilde{p}_{ij}^\mu = Q^\mu - \tilde{p}_k^\mu, \quad (4.5)$$

with λ the Källén function

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz. \quad (4.6)$$

There are three cases to consider

- $Q \rightarrow Q(\mathbf{p}_i, \lambda_i) + g(\mathbf{p}_j, \lambda_j)$, with $m_i = m_{ij} = m_Q$, $m_j = 0$ and

$$\begin{aligned} \langle \lambda' | \mathbf{V}_{Qg,k} | \lambda \rangle &= 8\pi\alpha_s C_F \tag{4.7} \\ &\times \left\{ \frac{\delta_{\lambda\lambda_i}}{1 - \tilde{z}_i(1 - y_{ij,k})} - \delta'_{\lambda_i\lambda_j} \frac{\tilde{v}_{ij,k}}{v_{ij,k}} \left[\delta_{\lambda\lambda_i}(1 + \tilde{z}_i) + \delta_{\lambda\lambda_j} \frac{m_Q^2}{p_i p_j} \right] \right. \\ &\left. - \frac{\tilde{v}_{ij,k}}{v_{ij,k}} \left[(\delta_{\lambda\lambda_i} \delta_{\lambda_i\lambda_j} - \delta'_{\lambda\lambda_i} \delta'_{\lambda_i\lambda_j}) \frac{1}{\tilde{z}_i} + \delta'_{\lambda_i\lambda_j} (\delta_{\lambda\lambda_i} - \delta_{\lambda\lambda_j}) \tilde{z}_i \right] \frac{m_Q^2}{2p_i p_j} \right\} \delta_{\lambda'\lambda}; \end{aligned}$$

- $g \rightarrow Q(\mathbf{p}_i, \lambda_i) + \bar{Q}(\mathbf{p}_j, \lambda_j)$, with $m_i = m_j = m_Q$, $m_{ij} = 0$ and

$$\begin{aligned} \langle \lambda' | \mathbf{V}_{Q\bar{Q},k} | \lambda \rangle &= 8\pi\alpha_s T_R \tag{4.8} \\ &\times \frac{1}{v_{ij,k}} \left\{ \delta_{\lambda'\lambda} \left[\frac{\delta'_{\lambda_i\lambda_j}}{2} - \frac{\kappa}{2} \left(z_+ z_- - \frac{m_Q^2}{(p_i + p_j)^2} \right) \right] \right. \\ &- \frac{2\delta'_{\lambda_i\lambda_j}}{(p_i + p_j)^2} \left[\tilde{z}_i^{(m)} p_i^\mu - \tilde{z}_j^{(m)} p_j^\mu \right] \left[\tilde{z}_i^{(m)} p_i^\nu - \tilde{z}_j^{(m)} p_j^\nu \right] \epsilon_\mu^*(\tilde{p}_{ij}, \lambda') \epsilon_\nu(\tilde{p}_{ij}, \lambda) \\ &+ \delta_{\lambda'\lambda} \left[\delta'_{\lambda_i\lambda_j} \left(\delta_{\lambda\lambda_i} \tilde{z}_i + \delta_{\lambda\lambda_j} \tilde{z}_j - \frac{1}{2} \right) \right. \\ &\left. \left. + \left((\delta_{\lambda\lambda_i} - \delta'_{\lambda_i\lambda_j}) \frac{1}{\tilde{z}_i} + (\delta_{\lambda\lambda_j} - \delta'_{\lambda_i\lambda_j}) \frac{1}{\tilde{z}_j} \right) \frac{m_Q^2}{(p_i + p_j)^2} \right] \right\}; \end{aligned}$$

- $g \rightarrow g(\mathbf{p}_i, \lambda_i) + g(\mathbf{p}_j, \lambda_j)$, with $m_i = m_j = m_{ij} = 0$ and

$$\begin{aligned} \langle \lambda' | \mathbf{V}_{gg,k} | \lambda \rangle &= 8\pi\alpha_s C_A \tag{4.9} \\ &\times \left\{ \delta_{\lambda'\lambda} \left[\frac{\delta_{\lambda\lambda_i}}{1 - \tilde{z}_i(1 - y_{ij,k})} + \frac{\delta_{\lambda\lambda_j}}{1 - \tilde{z}_j(1 - y_{ij,k})} - \frac{2\delta'_{\lambda_i\lambda_j} - \kappa z_+ z_- / 2}{v_{ij,k}} \right] \right. \\ &+ \frac{\delta'_{\lambda_i\lambda_j}}{v_{ij,k}} \frac{1}{p_i p_j} \left[\tilde{z}_i^{(m)} p_i^\mu - \tilde{z}_j^{(m)} p_j^\mu \right] \left[\tilde{z}_i^{(m)} p_i^\nu - \tilde{z}_j^{(m)} p_j^\nu \right] \epsilon_\mu^*(\tilde{p}_{ij}, \lambda') \epsilon_\nu(\tilde{p}_{ij}, \lambda) \\ &\left. + \frac{\delta_{\lambda'\lambda}}{v_{ij,k}} \left[\delta_{\lambda\lambda_i}(1 - 2\tilde{z}_i) + \delta_{\lambda\lambda_j}(1 - 2\tilde{z}_j) \right] \right\}; \end{aligned}$$

where $\delta'_{\lambda_i\lambda_j} = 1 - \delta_{\lambda_i\lambda_j}$. In each case, the content of the last square bracket vanishes upon summation over helicities of the emitter pair. The remaining terms have exactly the same structure as in [65] and are only modified by delta's in helicity. The variables \tilde{z}_i , \tilde{z}_j and $y_{ij,k}$ are defined as follows

$$\tilde{z}_i = 1 - \tilde{z}_j = \frac{p_i p_k}{p_i p_k + p_j p_k}, \tag{4.10}$$

$$y_{ij,k} = \frac{p_i p_j}{p_i p_j + p_i p_k + p_j p_k}. \tag{4.11}$$

The gluon splitting requires additionally

$$\tilde{z}_i^{(m)} = \tilde{z}_i - \frac{1}{2}(1 - v_{ij,k}), \quad \tilde{z}_j^{(m)} = \tilde{z}_j - \frac{1}{2}(1 - v_{ij,k}), \tag{4.12}$$

where $v_{ij,k}$ is the relative velocity between the emitter pair with momentum $p_i + p_j$ and the spectator with momentum p_k , and can be expressed as

$$v_{ij,k} = \frac{\sqrt{[2\mu_k^2 + (1 - \mu_i^2 - \mu_j^2 - \mu_k^2)(1 - y_{ij,k})]^2 - 4\mu_k^2}}{(1 - \mu_i^2 - \mu_j^2 - \mu_k^2)(1 - y_{ij,k})}, \quad (4.13)$$

with

$$\mu_n = m_n / \sqrt{Q^2}. \quad (4.14)$$

Similarly, the relative velocity between the emitter with momentum \tilde{p}_{ij} and the spectator with momentum \tilde{p}_k can be written as

$$\tilde{v}_{ij,k} = \frac{\sqrt{\lambda(1, \mu_{ij}^2, \mu_k^2)}}{1 - \mu_{ij}^2 - \mu_k^2}. \quad (4.15)$$

Moreover, the terms proportional to the free parameter κ require the introduction of

$$z_{\pm}(y_{ij,k}) = \frac{2\mu_i^2 + (1 - \mu_i^2 - \mu_j^2 - \mu_k^2)y_{ij,k}}{2[\mu_i^2 + \mu_j^2 + (1 - \mu_i^2 - \mu_j^2 - \mu_k^2)y_{ij,k}]} (1 \pm v_{ij,i}v_{ij,k}), \quad (4.16)$$

where $v_{ij,i}$ is the relative velocity between $p_i + p_j$ and p_i

$$v_{ij,i} = \frac{\sqrt{(1 - \mu_i^2 - \mu_j^2 - \mu_k^2)^2 y_{ij,k}^2 - 4\mu_i^2 \mu_j^2}}{(1 - \mu_i^2 - \mu_j^2 - \mu_k^2)y_{ij,k} + 2\mu_i^2}. \quad (4.17)$$

At this point it is important to comment on the freedom in the modification of the original formulae of [65]. Since we only require that these be recovered upon summation over helicities, arbitrary factors can be introduced as long as they do not spoil the collinear and/or soft limits. We have used this freedom to make the terms, which vanish upon helicity summation, resemble those that do not. Thus, for example, we have introduced the coefficient $\tilde{v}_{ij,k}/v_{ij,k}$ in front of the last square bracket in eq. (4.7), and similarly $1/v_{ij,k}$ in eq. (4.9). A similar freedom exists in the treatment of the parameter κ . As it represents a soft/collinear safe modification, it is unconstrained in the singular limits, and we are free to distribute it among different polarizations as we please. Here, we have chosen an even distribution among the four possible combinations.

4.2 Final-state emitter and initial-state spectator

The single dipole contribution is

$$\begin{aligned} \mathcal{D}_{ij}^a(\dots, \{p_i, \lambda_i\}, \dots, \{p_j, \lambda_j\}, \dots; \{p_a, \lambda_a\}, \dots) &= -\frac{1}{(p_i + p_j)^2 - m_{ij}^2} \frac{1}{x_{ij,a}} \\ &\times \sum_{\lambda', \lambda} \langle \dots, \{\tilde{p}_{ij}, \lambda'\}, \dots; \{\tilde{p}_a, \lambda_a\}, \dots | \frac{\mathbf{T}_a \cdot \mathbf{T}_{ij}}{\mathbf{T}_{ij}^2} \mathbf{V}_{ij}^a | \dots, \{\tilde{p}_{ij}, \lambda\}, \dots; \{\tilde{p}_a, \lambda_a\}, \dots \rangle, \end{aligned} \quad (4.18)$$

where

$$x_{ij,a} = \frac{p_a p_i + p_a p_j - p_i p_j + \frac{1}{2}(m_{ij}^2 - m_i^2 - m_j^2)}{p_a p_i + p_a p_j}, \quad (4.19)$$

and the momentum remapping is given by

$$\tilde{p}_a^\mu = x_{ij,a} p_a^\mu, \quad (4.20)$$

$$\tilde{p}_{ij}^\mu = p_i^\mu + p_j^\mu - (1 - x_{ij,a}) p_a^\mu. \quad (4.21)$$

There are three cases to consider

- $Q \rightarrow Q(\mathbf{p}_i, \lambda_i) + g(\mathbf{p}_j, \lambda_j)$, with $m_i = m_{ij} = m_Q$, $m_j = 0$ and

$$\begin{aligned} \langle \lambda' | \mathbf{V}_{Qg}^a | \lambda \rangle &= 8\pi\alpha_s C_F \quad (4.22) \\ &\times \left\{ \frac{\delta_{\lambda\lambda_i}}{1 - \tilde{z}_i + (1 - x_{ij,a})} - \delta'_{\lambda_i\lambda_j} \left(\delta_{\lambda\lambda_i} (1 + \tilde{z}_i) + \delta_{\lambda\lambda_j} \frac{m_Q^2}{p_i p_j} \right) \right. \\ &\left. - \left[(\delta_{\lambda\lambda_i} \delta_{\lambda_i\lambda_j} - \delta'_{\lambda\lambda_i} \delta'_{\lambda_i\lambda_j}) \frac{1}{\tilde{z}_i} + \delta'_{\lambda_i\lambda_j} (\delta_{\lambda\lambda_i} - \delta_{\lambda\lambda_j}) \tilde{z}_i \right] \frac{m_Q^2}{2p_i p_j} \right\} \delta_{\lambda'\lambda}; \end{aligned}$$

- $g \rightarrow Q(\mathbf{p}_i, \lambda_i) + \bar{Q}(\mathbf{p}_j, \lambda_j)$, with $m_i = m_j = m_Q$, $m_{ij} = 0$ and

$$\begin{aligned} \langle \lambda' | \mathbf{V}_{Q\bar{Q}}^a | \lambda \rangle &= 8\pi\alpha_s T_R \quad (4.23) \\ &\times \left\{ \frac{\delta_{\lambda'\lambda} \delta'_{\lambda_i\lambda_j}}{2} - \frac{2\delta'_{\lambda_i\lambda_j}}{(p_i + p_j)^2} \left[\tilde{z}_i p_i^\mu - \tilde{z}_j p_j^\mu \right] \left[\tilde{z}_i p_i^\nu - \tilde{z}_j p_j^\nu \right] \epsilon_\mu^*(\tilde{p}_{ij}, \lambda') \epsilon_\nu(\tilde{p}_{ij}, \lambda) \right. \\ &+ \delta_{\lambda'\lambda} \left[\delta'_{\lambda_i\lambda_j} \left(\delta_{\lambda\lambda_i} \tilde{z}_i + \delta_{\lambda\lambda_j} \tilde{z}_j - \frac{1}{2} \right) \right. \\ &\left. \left. + \left((\delta_{\lambda\lambda_i} - \delta'_{\lambda_i\lambda_j}) \frac{1}{\tilde{z}_i} + (\delta_{\lambda\lambda_j} - \delta'_{\lambda_i\lambda_j}) \frac{1}{\tilde{z}_j} \right) \frac{m_Q^2}{(p_i + p_j)^2} \right] \right\}; \end{aligned}$$

- $g \rightarrow g(\mathbf{p}_i, \lambda_i) + g(\mathbf{p}_j, \lambda_j)$, with $m_i = m_j = m_{ij} = 0$ and

$$\begin{aligned} \langle \lambda' | \mathbf{V}_{gg}^a | \lambda \rangle &= 8\pi\alpha_s C_A \quad (4.24) \\ &\times \left\{ \delta_{\lambda'\lambda} \left(\frac{\delta_{\lambda\lambda_i}}{1 - \tilde{z}_i + (1 - x_{ij,a})} + \frac{\delta_{\lambda\lambda_j}}{1 - \tilde{z}_j + (1 - x_{ij,a})} - 2\delta'_{\lambda_i\lambda_j} \right) \right. \\ &+ \frac{\delta'_{\lambda_i\lambda_j}}{p_i p_j} \left[\tilde{z}_i p_i^\mu - \tilde{z}_j p_j^\mu \right] \left[\tilde{z}_i p_i^\nu - \tilde{z}_j p_j^\nu \right] \epsilon_\mu^*(\tilde{p}_{ij}, \lambda') \epsilon_\nu(\tilde{p}_{ij}, \lambda) \\ &\left. + \delta_{\lambda'\lambda} \left[\delta_{\lambda\lambda_i} (1 - 2\tilde{z}_i) + \delta_{\lambda\lambda_j} (1 - 2\tilde{z}_j) \right] \right\}; \end{aligned}$$

where

$$\tilde{z}_i = \frac{p_a p_i}{p_a p_i + p_a p_j}, \quad \tilde{z}_j = \frac{p_a p_j}{p_a p_i + p_a p_j}. \quad (4.25)$$

As in the previous case, terms in the last square bracket vanish upon summation over emitter pair helicities.

4.3 Initial-state emitter and final-state spectator

The single dipole contribution is

$$\mathcal{D}_j^{ai}(\dots, \{p_i, \lambda_i\}, \dots, \{p_j, \lambda_j\}, \dots; \{p_a, \lambda_a\}, \dots) = -\frac{1}{2p_a p_i} \frac{1}{x_{ij,a}} \quad (4.26)$$

$$\times \sum_{\lambda', \lambda} \langle \dots, \{\tilde{p}_j, \lambda_j\}, \dots; \{\tilde{p}_{ai}, \lambda'\}, \dots | \frac{\mathbf{T}_j \cdot \mathbf{T}_{ai}}{T_{ai}^2} \mathbf{V}_j^{ai} | \dots, \{\tilde{p}_j, \lambda_j\}, \dots; \{\tilde{p}_{ai}, \lambda\}, \dots \rangle,$$

where

$$x_{ij,a} = \frac{p_a p_i + p_a p_j - p_i p_j}{p_a p_i + p_a p_j}, \quad (4.27)$$

and the momentum remapping is given by

$$\tilde{p}_j^\mu = p_i^\mu + p_j^\mu - (1 - x_{ij,a}) p_a^\mu, \quad (4.28)$$

$$\tilde{p}_{ai}^\mu = x_{ij,a} p_a^\mu. \quad (4.29)$$

As implicitly assumed above, we require both emitter masses to vanish, i.e. $m_i = m_j = m_{ij} = 0$. There are four cases to consider

- $q(p_a, \lambda_a) \rightarrow g(p_i, \lambda_i) + q$, with

$$\langle \lambda' | \mathbf{V}_j^{qq} | \lambda \rangle = 8\pi\alpha_s C_F \quad (4.30)$$

$$\times \left\{ \frac{1}{1 - x_{ij,a} + u_i} - \delta'_{\lambda_a \lambda_i} (1 + x_{ij,a}) \right\} \delta_{\lambda' \lambda} \delta_{\lambda \lambda_a};$$

- $g(p_a, \lambda_a) \rightarrow \bar{q}(p_i, \lambda_i) + q$, with

$$\langle \lambda' | \mathbf{V}_j^{g\bar{q}} | \lambda \rangle = 8\pi\alpha_s T_R \quad (4.31)$$

$$\times \left\{ \delta_{\lambda_a \lambda_i} (1 - 2x_{ij,a}(1 - x_{ij,a})) + [1 - 2\delta_{\lambda_a \lambda_i}] x_{ij,a}^2 \right\} \delta_{\lambda' \lambda} \delta'_{\lambda \lambda_i};$$

- $q(p_a, \lambda_a) \rightarrow q(p_i, \lambda_i) + g$, with

$$\langle \lambda' | \mathbf{V}_j^{qq} | \lambda \rangle = 8\pi\alpha_s C_F \quad (4.32)$$

$$\times \left\{ \delta_{\lambda' \lambda} \delta'_{\lambda \lambda_a} \delta_{\lambda_a \lambda_i} x_{ij,a} \right.$$

$$+ \delta_{\lambda_a \lambda_i} \frac{1 - x_{ij,a}}{x_{ij,a}} \frac{u_i(1 - u_i)}{p_i p_j} \left(\frac{p_i^\mu}{u_i} - \frac{p_j^\mu}{1 - u_i} \right) \left(\frac{p_i^\nu}{u_i} - \frac{p_j^\nu}{1 - u_i} \right) \epsilon_\mu(\tilde{p}_{ai}, \lambda') \epsilon_\nu^*(\tilde{p}_{ai}, \lambda)$$

$$\left. + \delta_{\lambda' \lambda} \delta_{\lambda_a \lambda_i} [2\delta_{\lambda \lambda_a} - 1] \right\};$$

- $g(p_a, \lambda_a) \rightarrow g(p_i, \lambda_i) + g$, with

$$\begin{aligned}
 \langle \lambda' | \mathbf{V}^{gg} | \lambda \rangle &= 8\pi\alpha_s C_A \tag{4.33} \\
 &\times \left\{ \delta_{\lambda'\lambda} \left[\frac{\delta_{\lambda\lambda_a}}{1 - x_{ij,a} + u_i} + \delta'_{\lambda\lambda_i} (-1 + x_{ij,a}(1 - x_{ij,a})) \right] \right. \\
 &+ \delta_{\lambda_a\lambda_i} \frac{1 - x_{ij,a}}{x_{ij,a}} \frac{u_i(1 - u_i)}{p_i p_j} \left(\frac{p_i^\mu}{u_i} - \frac{p_j^\mu}{1 - u_i} \right) \left(\frac{p_i^\nu}{u_i} - \frac{p_j^\nu}{1 - u_i} \right) \epsilon_\mu(\tilde{p}_{ai}, \lambda') \epsilon_\nu^*(\tilde{p}_{ai}, \lambda) \\
 &\left. + \delta_{\lambda'\lambda} [(\delta_{\lambda\lambda_a} - \delta'_{\lambda\lambda_i}) + (\delta_{\lambda_a\lambda_i} - \delta_{\lambda\lambda_a}) 2x_{ij,a}] \right\};
 \end{aligned}$$

where

$$u_i = \frac{p_i p_a}{p_i p_a + p_j p_a}. \tag{4.34}$$

4.4 Initial-state emitter and initial-state spectator

The single dipole contribution is

$$\begin{aligned}
 \mathcal{D}^{ai,b}(\dots, \{p_i, \lambda_i\}, \dots; \{p_a, \lambda_a\}, \{p_b, \lambda_b\}) &= -\frac{1}{2p_a p_i} \frac{1}{x_{i,ab}} \tag{4.35} \\
 &\times \sum_{\lambda', \lambda} \langle \dots; \{\tilde{p}_{ai}, \lambda'\}, \{p_b, \lambda_b\} | \frac{\mathbf{T}_b \cdot \mathbf{T}_{ai}}{\mathbf{T}_{ai}^2} \mathbf{V}^{ai,b} | \dots; \{\tilde{p}_{ai}, \lambda\}, \{p_b, \lambda_b\} \rangle,
 \end{aligned}$$

where

$$x_{i,ab} = \frac{p_a p_b - p_i p_a - p_i p_b}{p_a p_b}, \tag{4.36}$$

and the momentum remapping is given by

$$\tilde{p}_{ai}^\mu = x_{i,ab} p_a^\mu, \tag{4.37}$$

$$\tilde{k}_j^\mu = k_j^\mu - \frac{2k_j \cdot (K + \tilde{K})}{(K + \tilde{K})^2} (K + \tilde{K})^\mu + \frac{2k_j \cdot K}{K^2} \tilde{K}^\mu, \tag{4.38}$$

where the index j runs over all final states and the momenta K^μ and \tilde{K}^μ are defined by

$$\begin{aligned}
 K^\mu &= p_a^\mu + p_b^\mu - p_i^\mu, \\
 \tilde{K}^\mu &= \tilde{p}_{ai}^\mu + p_b^\mu.
 \end{aligned} \tag{4.39}$$

We require again both emitter masses to vanish, i.e. $m_i = m_j = m_{ij} = 0$. There are four cases to consider

- $q(p_a, \lambda_a) \rightarrow g(p_i, \lambda_i) + q$, with

$$\begin{aligned}
 \langle \lambda' | \mathbf{V}^{qg,b} | \lambda \rangle &= 8\pi\alpha_s C_F \tag{4.40} \\
 &\times \left\{ \frac{1}{1 - x_{i,ab}} - \delta'_{\lambda_a\lambda_i} (1 + x_{i,ab}) \right\} \delta_{\lambda'\lambda} \delta_{\lambda\lambda_a};
 \end{aligned}$$

- $g(p_a, \lambda_a) \rightarrow \bar{q}(p_i, \lambda_i) + q$, with

$$\begin{aligned} \langle \lambda' | \mathbf{V}^{g\bar{q},b} | \lambda \rangle &= 8\pi\alpha_s T_R & (4.41) \\ &\times \left\{ \delta_{\lambda_a \lambda_i} (1 - 2x_{i,ab}(1 - x_{i,ab})) + [1 - 2\delta_{\lambda_a \lambda_i}] x_{i,ab}^2 \right\} \delta_{\lambda' \lambda} \delta'_{\lambda \lambda_i}; \end{aligned}$$

- $q(p_a, \lambda_a) \rightarrow q(p_i, \lambda_i) + g$, with

$$\begin{aligned} \langle \lambda' | \mathbf{V}^{qq,b} | \lambda \rangle &= 8\pi\alpha_s C_F & (4.42) \\ &\times \left\{ \delta_{\lambda' \lambda} \delta'_{\lambda \lambda_a} \delta_{\lambda_a \lambda_i} x_{i,ab} \right. \\ &+ \delta_{\lambda_a \lambda_i} \frac{1 - x_{i,ab}}{x_{i,ab}} \frac{p_a \cdot p_b}{p_i \cdot p_a p_i \cdot p_b} \left(p_i^\mu - \frac{p_i p_a}{p_b p_a} p_b^\mu \right) \left(p_i^\nu - \frac{p_i p_a}{p_b p_a} p_b^\nu \right) \\ &\left. \times \epsilon_\mu(\tilde{p}_{ai}, \lambda') \epsilon_\nu^*(\tilde{p}_{ai}, \lambda) + \delta_{\lambda' \lambda} \delta_{\lambda_a \lambda_i} [2\delta_{\lambda \lambda_a} - 1] \right\}; \end{aligned}$$

- $g(p_a, \lambda_a) \rightarrow g(p_i, \lambda_i) + g$, with

$$\begin{aligned} \langle \lambda' | \mathbf{V}^{gg,b} | \lambda \rangle &= 8\pi\alpha_s C_A & (4.43) \\ &\times \left\{ \delta_{\lambda' \lambda} \left[\frac{\delta_{\lambda \lambda_a}}{1 - x_{i,ab}} + \delta'_{\lambda \lambda_i} (-1 + x_{i,ab}(1 - x_{i,ab})) \right] \right. \\ &+ \delta_{\lambda_a \lambda_i} \frac{1 - x_{i,ab}}{x_{i,ab}} \frac{p_a \cdot p_b}{p_i \cdot p_a p_i \cdot p_b} \left(p_i^\mu - \frac{p_i p_a}{p_b p_a} p_b^\mu \right) \left(p_i^\nu - \frac{p_i p_a}{p_b p_a} p_b^\nu \right) \\ &\left. \times \epsilon_\mu(\tilde{p}_{ai}, \lambda') \epsilon_\nu^*(\tilde{p}_{ai}, \lambda) + \delta_{\lambda' \lambda} [(\delta_{\lambda \lambda_a} - \delta'_{\lambda \lambda_i}) + (\delta_{\lambda_a \lambda_i} - \delta_{\lambda \lambda_a}) 2x_{i,ab}] \right\}. \end{aligned}$$

5 Implementation and example results

As part of the present study, we have implemented the complete formalism given in the previous section within the framework of HELAC-PHEGAS. The software can be obtained from the HELAC-PHEGAS web page [85]. The main features can be summarized as follows

1. Arbitrary processes

We use the matrix element generator of HELAC-PHEGAS, which means that any process, which can be calculated by this generator is also accessible to the dipole subtraction software. The only limitation is given by the models implemented. Currently, the full Standard Model, both electroweak and QCD, is available. Let us stress that the subtraction is only applied to partons, and the software is therefore devised for NLO QCD corrections. The observables are specified by user defined jet functions and histogramming routines. The present implementation contains a built in k_T jet algorithm.

2. Massive and massless external states

We have implemented the formulae as given in the previous section. This allows us to treat massive and massless partons on the same footing. As noted in the previous section, we assume the initial state partons to be massless.

3. Helicity sampling for partons

The integration over the phase space is done in three stages. At first, the phase space is optimized using multi-channel methods. In this phase, the user can either evaluate the subtracted matrix elements with full summation, or use a flat helicity Monte Carlo, where all helicity configurations occur with the same frequency. Since this part is only used to determine the weights of the different channels we recommend to run in the latter mode, which is fast and gives exactly the same results as full summation. In a second stage, the phase space measure is fixed and the helicity configuration sampling weights are determined by evaluating the subtracted matrix element for all helicity configurations. The optimization consists of a standard minimization of the variance [80], but is as slow as full summation (in fact, this *is* full summation). We recommend to run this phase on a few hundreds up to a few thousand points. The optimum depends strongly on the number of helicity configurations. Finally, in the last stage, the matrix element is evaluated with helicity sampling, which updates itself at user specified intervals in the number of accepted events. These intervals should be long enough, to allow the channels which have a low weight to accumulate enough events. We recommend a number of the order of ten thousand. All the parameters can be changed by the user. Obviously, one can make a complete run using full helicity summation or flat Monte Carlo. As with any simulation, some experimentation is needed to obtain best results.

4. Random polarizations for non-partons

Non-partons can be treated in two different ways as far as their polarization is concerned. The user can either require random phases as described in section 2, or treat non-partons on the same footing as partons. We recommend to use random polarizations.

5. Restrictions on the subtraction phase space

We have implemented a restriction on the phase space of the subtraction as proposed in [6]. This amounts to only including dipole subtraction terms, which satisfy the following criteria

- (a) final-final dipoles

$$y_{ij,k} < \alpha_{\max}^{FF} ; \tag{5.1}$$

- (b) final-initial dipoles

$$1 - x_{ij,a} < \alpha_{\max}^{FI} ; \tag{5.2}$$

(c) initial-final dipoles

$$u_i < \alpha_{\max}^{IF} ; \tag{5.3}$$

(d) initial-initial dipoles

$$\tilde{v}_i \equiv \frac{p_a p_i}{p_a p_b} < \alpha_{\max}^{II} ; \tag{5.4}$$

where the occurring variables have been defined in the previous section. The four α_{\max} parameters can be freely chosen by the user. A value of 1 amounts to no restriction. Best results are obviously obtained with small α_{\max} , since the matrix element for this setting behaves largely as the real emission alone and the phase space optimizer has been designed to emulate such behavior. Besides the phase space restriction, we have also included a technical cut. Using the above formulae one can evaluate a minimal value of the four parameters given on the left hand side for all dipoles. Let us denote it by α_{\min} . We reject a phase space point completely if

$$\alpha_{\min} < \alpha_{\text{cut}} . \tag{5.5}$$

The value of α_{cut} should be specified by the user. Let us stress that the technical cut is necessary to avoid numerical instabilities in the cancellation between real emission and the dipoles. However, there are also cutoffs in the phase space generator, which are needed to avoid numerical instabilities in the generator itself.

6. Phase space integration

The integration is completely controlled by PHEGAS [74], which is a multi-channel phase space generator based on Feynman graphs. The only modification, which was necessary was the simultaneous treatment of both positive and negative weights. In fact, the user can specify whether only positive, negative or both weights should be included. This does not modify the histograms, however, which always contain the full result.

We should mention that in the early stages of the development, we have adapted the kinematics remapping routine from MCFM [86]. Although, a minute part of the final code, it was important for debugging to be able to rely on the correctness of the kinematics.

This software has been tested against MADDIPOLE [87]. We would like, however, to point out that instead of comparing arbitrary processes, we concentrated on the individual parts of the code. In fact, the only sensitive code, which cannot be tested against the official version of HELAC-PHEGAS is contained in the kinematics remapping, the color correlators and the dipole subtraction formulae themselves. In consequence, to ensure the correctness of the implementation it was sufficient to compare the latter for all independent cases. These are summarized in table 1. Of course, we could only compare the dipole subtraction terms after summation over helicities. The expressions for independent helicity configurations have been tested by checking for cancellation in the appropriate limits.

\mathcal{E}_0 - massless emitter, \mathcal{S}_0 - massless spectator, \mathcal{E}_M - massive emitter, \mathcal{S}_M - massive spectator, \mathcal{E}_I - initial state emitter, \mathcal{E}_F - final state emitter, \mathcal{S}_I - initial state spectator, \mathcal{S}_F - final state spectator, \checkmark - check, \blacksquare - does not occur.

	$\mathcal{E}_0/\mathcal{S}_0$	$\mathcal{E}_0/\mathcal{S}_M$	$\mathcal{E}_M/\mathcal{S}_0$	$\mathcal{E}_M/\mathcal{S}_M$
$\mathcal{E}_I/\mathcal{S}_I$				
$g \rightarrow gg$	\checkmark	\blacksquare	\blacksquare	\blacksquare
$g \rightarrow qq$	\checkmark	\blacksquare	\blacksquare	\blacksquare
$q \rightarrow qq$	\checkmark	\blacksquare	\blacksquare	\blacksquare
$q \rightarrow gq$	\checkmark	\blacksquare	\blacksquare	\blacksquare
$\mathcal{E}_I/\mathcal{S}_F$				
$g \rightarrow gg$	\checkmark	\checkmark	\blacksquare	\blacksquare
$g \rightarrow qq$	\checkmark	\checkmark	\blacksquare	\blacksquare
$q \rightarrow qq$	\checkmark	\checkmark	\blacksquare	\blacksquare
$q \rightarrow gq$	\checkmark	\checkmark	\blacksquare	\blacksquare

	$\mathcal{E}_0/\mathcal{S}_0$	$\mathcal{E}_0/\mathcal{S}_M$	$\mathcal{E}_M/\mathcal{S}_0$	$\mathcal{E}_M/\mathcal{S}_M$
$\mathcal{E}_F/\mathcal{S}_I$				
$g \rightarrow gg$	\checkmark	\blacksquare	\blacksquare	\blacksquare
$g \rightarrow qq$	\checkmark	\blacksquare	\checkmark	\blacksquare
$q \rightarrow qq$	\checkmark	\blacksquare	\checkmark	\blacksquare
$q \rightarrow gq$	\checkmark	\blacksquare	\checkmark	\blacksquare
$\mathcal{E}_F/\mathcal{S}_F$				
$g \rightarrow gg$	\checkmark	\checkmark	\blacksquare	\blacksquare
$g \rightarrow qq$	\checkmark	\checkmark	\checkmark	\checkmark
$q \rightarrow qq$	\checkmark	\checkmark	\checkmark	\checkmark
$q \rightarrow gq$	\checkmark	\checkmark	\checkmark	\checkmark

Table 1. Independent dipole splitting formulae, which need to be tested in order to ensure the correctness of the code. In the splitting description, e.g. $g \rightarrow gg$, the left hand side particle always denotes the virtual state.

PROCESS	REAL EMISSION + DIPOLES [msec]	REAL EMISSION [msec]	NR OF DIPOLES
$gg \rightarrow ggg$	3.8	1.0	27
$gg \rightarrow gggg$	8.5	2.6	56
$gg \rightarrow ggggg$	300	42	100
$u\bar{d} \rightarrow W^+ gggg$	9.3	2.4	56
$gg \rightarrow t\bar{t}b\bar{b}g$	12	2.9	55

Table 2. The CPU time needed to evaluate the real emission matrix element together with all of the dipole subtraction terms per phase-space point (this corresponds to $\alpha_{\max} = 1$). All numbers have been obtained on an Intel 2.53 GHz Core 2 Duo processor with the Intel Fortran compiler using the *-fast* option.

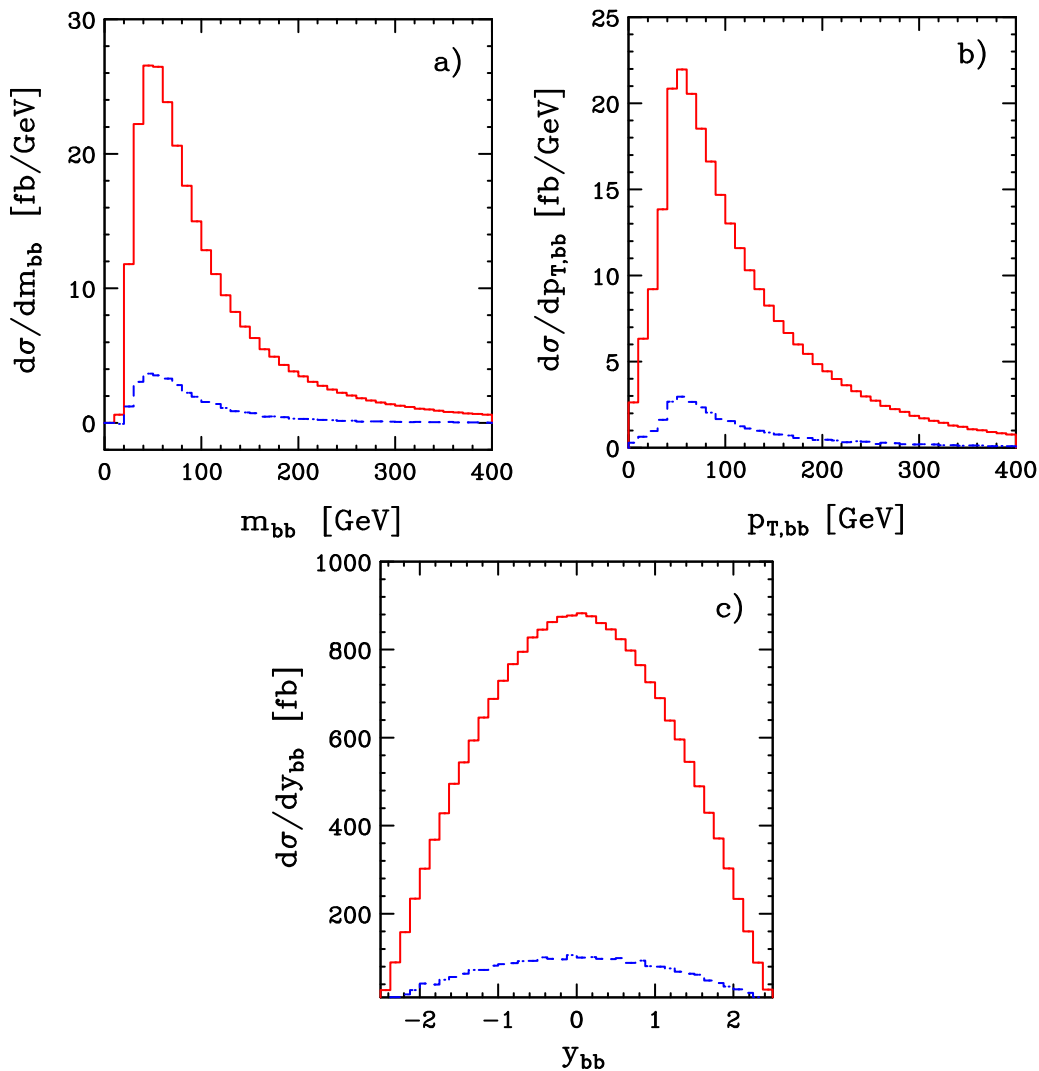


Figure 1. Distribution of the invariant mass $m_{b\bar{b}}$ of the bottom-anti-bottom pair (a), distribution in the transverse momentum $p_{T,b\bar{b}}$ of the bottom-anti-bottom pair (b) and distribution in the rapidity $y_{b\bar{b}}$ of the bottom-anti-bottom pair (c) for $pp(gg) \rightarrow t\bar{t}b\bar{b}g + X$ at the LHC for two different values of α_{\max} : 0.01 (red solid curve) and 1 (blue dashed curve).

Additionally, in table 2 we have presented the measured time needed to evaluate the real emission matrix element and the subtraction terms. We note that the inclusion of the full set of subtraction terms slows down the computation by a factor of about three to four in most cases. However, since there is a restriction in the subtraction phase space, we expect (and indeed observe in practice) that the true additional cost of the subtraction does not exceed the cost of the real emission itself. Therefore, an improvement in the speed of the calculation will not be obtained by replacing the dipole formalism by another subtraction. In fact, a further substantial speed up can be obtained by turning to Monte Carlo summation over color configurations. What is the source of the high cost of the

evaluation of the subtracted real radiation in comparison to leading order real radiation? It lies in the phase space integration. The integrand has now a much more complicated behavior, and therefore multi-channel optimization based on Feynman graphs does not lead to such a drastic improvement of the convergence.

As a final demonstration of the power of our implementation, we have performed a simulation of the $gg \rightarrow t\bar{t}b\bar{b}g$ subprocess, which is part of the complete NLO calculation of $pp \rightarrow t\bar{t}b\bar{b} + X$. We have used the same set up as [18]. In particular, we have taken $\sqrt{s} = 14$ TeV as center of mass energy and a top quark mass of $m_t = 172.6$ GeV. The b quark has been kept massless. In order to obtain jets, we have used the k_T algorithm [88–90] with jet recombination of partons with a pseudo-rapidity of $|\eta| < 5$ with $\Delta R < 0.8$. Additionally, we required the b jets to satisfy $p_{T,b} > 20$ GeV, $|y_b| < 2.5$. The phase space of the top quarks has not been restricted. The non-perturbative input of our computation has been given by the CTEQ6M PDFs [91, 92]. Figure 1 (a-c) contains the invariant mass, transverse momentum and rapidity distributions of the $b\bar{b}$ pair for $\alpha_{\max} = 0.01$ and $\alpha_{\max} = 1$ (all parameters set to the same value). The impact of the variation of α_{\max} is clearly visible. In fact, for both values, the shape resembles that of the leading order result, besides the normalization. On the other hand, for larger values of α_{\max} a relatively small negative dip for low invariant masses can be observed in the $m_{b\bar{b}}$ distribution. The latter is due to the strong enhancement of the dipole contributions for low $m_{b\bar{b}}$.

6 Conclusions

We have presented an extension of the dipole subtraction formalism to arbitrary helicity eigenstates. Not only did we provide appropriate formulae, but also a public implementation, which contains both the subtracted real radiation and integrated dipoles. The results of our tests show that with this software, calculations, which are of current phenomenological interest, can be performed fully automatically within tractable time. In the nearest future, we will apply this technology to the processes from the “NLO wishlist” [93] using the automated 1-loop extension of HELAC [41] for the virtual corrections.

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